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Methodology for Performance Analysis of Aerospace Vehicles Using the Laws of Thermodynamics

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Theory, methodology, and example applications are developed and shown for the systematic analysis of overall vehicle forces in terms of irreversibility and heat. The methodology presented involves analyzing and deconstructing vehicle forces using individual stream tubes as components within the overall fluid control volume in which the vehicle is embedded. This provides the capability for the complete fluid/thermodynamic "audit" of vehicle performance in terms of irreversibility, combustion (heating), and fluid dynamic flow turning and area change. Sample results are shown for a simplified hypersonic vehicle configuration modeled with constant specific heats and Rayleigh heating. The role of overall entropy generation and wake mixing processes in the production of vehicle forces in atmospheric flight is next discussed and clarified. Specifically, the direct analytical relationship between entropy, wake mixing processes, and overall force production for the vehicle is developed from fundamental considerations of the global control volume with inclusion of the wake in the analysis. This analysis is demonstrated using the same simplified high-speed configuration and is finally developed for the completely general problem of an aerospace vehicle with variable specific heats, thermal loading, variable composition, fuel injection, and chemical reaction.

Nomenclature

A	=	cross-sectional area, m^2
C_P	=	specific heat at constant pressure, J/kg-K
$\boldsymbol{F}$	=	force vector, N
$\boldsymbol{F}_{\text{bound(shear)}}$	=	viscous boundary force on side of stream tube, N
F_x, F_y, F_z	=	x, y, z Cartesian force components, N
H_f	=	heating value of fuel, J/kg
$\dot{H}_t$	=	total enthalpy flow rate, J/s
M	=	Mach number
$\dot{m}$	=	mass flow rate, kg/s
mw	=	molecular weight, kg/kmol
ncs	=	number of chemical species
P	=	static pressure, N/m^2
$\dot{Q}$	=	heat rate, J/s
R	=	gas constant, J/kg-K
$\dot{S}$	=	entropy flow rate, J/K-s
T	=	static temperature, K
u, v, w	=	x, y, z Cartesian velocity components, m/s
$\boldsymbol{V}$	=	velocity vector, m/s
$\dot{W}$	=	work rate, J/s
α_l	=	species l mass fraction
β	=	flow angularity, deg
δq	=	differential heat rate per mass, J/kg
$\hat{\eta}$	=	unit vector normal to surface, oriented outward from fluid
η_l	=	species l mole fraction
ρ	=	static density, kg/m^3

Subscripts

e	=	vehicle exit plane
i, 0	=	vehicle leading edge plane, freestream
inj	=	fuel-injection conditions
irr	=	irreversible process
l	=	chemical species
r, rev	=	reversible process
t	=	total condition
veh	=	vehicle
w	=	wake exit plane
w, wake	=	wake process
0	=	reference

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I. Introduction

HIGH-SPEED airbreathing atmospheric flight is dominated by 1) extremely large flow irreversibility associated with both internal (engine) and external (aerodynamic) fluid streams and 2) (particularly for higher flight Mach numbers) very small ratios of onboard (useful) energy to freestream energy. This results in small overall performance margins, especially as flight Mach number is increased. Current design and analysis methodologies for aerospace vehicles in all speed regimes (subsonic to hypersonic) are primarily based on individual component-level or subsystem-level approaches to characterizing, analyzing, and optimizing respective piecewise performances. The resulting overall vehicle performance is then essentially a product of the net effect of this nonsynergistic approach. Nevertheless, adequate overall designs can result, generally as trades aimed at maximizing overall vehicle performance are made within a parametrically driven top-down overall system integration effort. In this sense, some degree of synergy of outcome can be achieved in current methodologies. However, there is an urgent need for a comprehensive and formalized approach to characterizing the impact of irreversibility on aerospace vehicle design, evaluation, and optimization particularly for hypersonic vehicle design caused by the dominance of irreversibility in that regime. This approach must be at the system level as proposed by Moorhouse.[1]

Moorhouse articulated the energy-based principle that the mission of a flight vehicle be considered as the required "customer work" with all other "overhead" work minimized. A successful strategy that is based on this principle should then be based specifically on a common currency for performance characterization of all components and subsystems that contribute to fuel usage. To develop this strategy, it is believed that the explicit impact of flow irreversibility on overall vehicle performance as conventionally defined (i.e., vehicle forces) must be fully understood in order to fully assess current designs, optimize designs, or develop new designs. Surprisingly, this subject is (from the practical vehicle performance-based standpoint) relatively untouched if one considers the significant benefits that can result from a full realization of second-law-based strategies. Although the following theoretical and methodology development focuses specifically on the high-speed flight regime where the impact of irreversibility is extreme, the theory and methodology described are quite general in terms of speed regime, that is, concepts and techniques described should apply for flight regimes ranging from subsonic to hypersonic.

This work first describes methodology that yields a full force-based performance audit for an entire hypersonic aerospace vehicle in terms of explicit second-law characteristics. This is done by beginning from fundamental considerations of forces experienced by stream tubes and defining the functional characterization of the forces, that is, defining the basic quantities which actually contribute to the production of forces on a given stream tube. This understanding is then methodically extended to the analysis of force interactions experienced by the wetted surfaces of a vehicle in high-speed flight. The analysis is rigorously based on the overall control-volume equations descriptive of the flow in an appropriately defined global control volume or stream tube (itself composed of many individual stream tubes) that completely envelopes the vehicle and the vehicle zone of influence out to a suitably defined exit plane. The complete characterization of the reference reversible forces, which the vehicle would experience if irreversibility were removed in the given system, are formulated from first principles based on this same control volume approach—the global stream tube enveloping the vehicle being appropriately analyzed in terms of constituent streamtubes for a full audit (or accounting) of the impact of both irreversibility as well as heat interaction and flow nonuniformity and turning effects. A simplified vehicle example based on constant specific heats and inviscid flow is shown utilizing this methodology, although the methodology is completely general.

This is followed by theoretical development that directly relates the force experienced by the vehicle to the entire vehicle/wake-mixing process (the wake-mixing process referring to the relaxation of the various flows at the vehicle exit plane to complete equilibrium with each other and with the environment downstream of the vehicle). This is done by again examining the global control volume in which the vehicle is embedded within the atmosphere; however, with the inclusion of the wake mixing domain aft of the vehicle itself. The theoretical development and analysis presented here are complementary to the development/methodology for characterizing the reference reversible forces; together they provide a complete theoretical/practical framework for fully analyzing and understanding the impact of irreversibility and energy interactions on high-speed vehicle performance.

Early work done in this area includes that of Foa[2] who sketched the characterization of performance system thrust and propulsive losses in terms of entropy rise across a jet engine (for matched pressure across the engine and other simplifying assumptions); he termed this the "entropy method." This is similar to the more general work of Riggins,[3] who recently examined the thermodynamic spectrum of gas-turbine-engine performance from the standpoint of overall heat added, irreversibility (entropy production), and work exchange in the propulsive flowpath. Lewis[4] provided clarification regarding the role of the second law in providing a universal definition of the propulsive efficiency for an isolated engine. The concept of thrust or thrust-work potential (also called stream-thrust-based methods) for the performance characterization of high-speed ramjet and scramjet engines was first articulated by Curran and Craig[5]; considerable extension of their seminal work has been performed by Riggins et al.[6] and Riggins.[7] The use of these methods has enabled the complete characterization of the loss in scramjet engine thrust caused by irreversibility and has allowed the assessment of engine thrust losses in terms of irreversible loss mechanism and location. In a closely related development, the general concept of work availability as applied to aerospace jet engines (turbojets and turbofans) has been developed and utilized by Roth,[8,9] who has suggested the use of work availability as a common currency for engine design, evaluation, and optimization, generally without explicit consideration of entropy (second-law considerations) necessary. In addition, a significant amount of work has also been done in the area of applying conventional exergy (or availability) to the problem of aerospace vehicle design and evaluation (for example, see Clarke and Horlock[10] and Czysz and Murthy[11]). Availability is based on the assessment of the maximum reversible work as measured from a dead state and is attractive as a single-currency candidate, that is, it is well established and has an excellent track record for cyclic ground-based systems such as power plants. However, Riggins[12,13] has shown problems with conventional availability when directly applied to very simple jet engine optimization problems and has suggested a modification of exergy (called engine-based exergy), which essentially unifies it with the stream thrust concepts already discussed. The complete relationship between availability, entropy, and overall vehicle performance is derived and discussed in the latter sections of this paper in which the critical importance of the vehicle wake-mixing/equilibrium process is fully demonstrated.

The references just listed, which detail previous/related work in the area of performance assessment, are specific to the propulsion systems for flight vehicles. There has been significantly less work involving the development and use of entropy-based methods for external aerodynamic design, evaluation, and optimization. An early reference that correctly incorporates the second-law impact of the wake process for the drag on a base aerodynamic shape without energy interactions or mass addition is found in the textbook by Oswatitsch.[14] Greene[15] examined the role of entropy for induced drag minimization on low-speed airfoils using concepts related to this earlier work. Roth[16] and Roth and Mavris[17] have usefully extended work potential methods to vehicle airframe and overall vehicle system loss management. Finally, Giles and Cummings[18] have presented work relating vehicle drag performance and wake processes which, although specifically focused on subsonic/transonic flight, is readily extended to other flight regimes.

II. Functional Dependence of Forces Experienced by Fluid in Streamtubes

Steady fluid flow around a high-speed aerospace vehicle and through its propulsion system can be analyzed rigorously in terms of multiple fluid stream tubes (each with quasi-one-dimensional fluid dynamics). In each stream tube originating upstream of the vehicle, fluid is processed from freestream conditions to an appropriately defined exit plane where the fluid effectively ceases to affect the vehicle and vice versa. Interactions between adjacent stream tubes and the vehicle wetted surfaces in terms of cross-sectional area changes, angularity (direction change), forces, and heat and work (energy) interactions drive stream-tube development and evolution. Additionally, fuel/mass addition to stream tubes inducted into the propulsion system takes place as well as diffusive mixing of disparate species between adjacent stream tubes downstream of mixing and combustion zones. Aft of the defined vehicle exit plane, further processing of the stream tubes takes place in the wake of the vehicle where there is a relaxation process back to a condition of displaced equilibrium. The interactions of the fluid in the stream tubes with the vehicle wetted surfaces (internal and external) give rise to forces, moments, and heat transfer on that vehicle, that is, determine vehicle performance. Of specific interest here is the overall force vector experienced by the vehicle and the development and demonstration of the ability to analyze the components of this overall force vector in terms of second-law characteristics and impacts.

Although a stream tube is arbitrarily defined in terms of its size (i.e., it can be defined as large in cross-sectional area as the fluid stream encompassing the entire vehicle or as small as the continuum approximation allows) and is itself composed of smaller stream tubes, it is obvious that the flowfield approximation fidelity and captured physics increase with greater density (number) of stream tubes defining the modeled flowfield (neglecting numerical issues). This

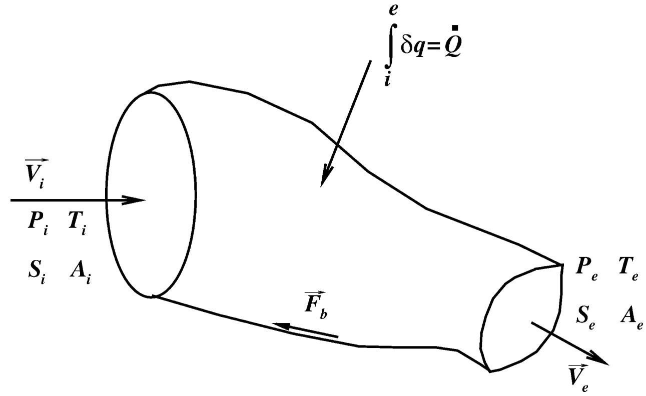

Fig. 1 Stream tube in flow (no shaft work).

is because at the most basic level of a stream tube, which is effectively modeled in a given flowfield, the quasi-one-dimensional approximation to the fluids within that stream tube is automatically applied. This is true for conventional multidimensional computational fluid dynamics as well, which, although not explicitly based on the stream-tube approach, nevertheless assumes in various fashions cell-averaged values between nodes.

Consider a stream tube (with quasi-one-dimensional characteristics) as sketched in Fig. 1. This stream tube is itself located in a larger (steady) fluid flow (or overall control volume or stream tube). For simplicity here, neglect any mass addition effects via injection or diffusive mixing although this assumption is not necessary in general. This stream tube is processed from some defined inlet plane i to exit plane e. It has an overall heat interaction rate $\dot{Q}$ with its surroundings [other stream tubes or vehicle surface(s)], which is progressive, that is,

$$\dot{Q} = \int_i^e \delta q$$

where δq is the differential heat interaction associated with a differential distance along the stream tube. Furthermore, it can have a force-based work rate interaction on the boundary, that is, as a result of flow in adjacent stream tubes having different velocities. This term will be as follows:

$$\int_i^e \mathrm{d}\boldsymbol{F}_{\text{bound(shear)}} \cdot \boldsymbol{V}_{\text{bound}}$$

where $\mathrm{d}\boldsymbol{F}_{\text{bound(shear)}}$ is the differential boundary (shear) force felt on the circumference of the streamtube at a location and $\boldsymbol{V}_{\text{bound}}$ is (properly) the velocity at the boundary. This implies an inner integration around the circumference of the stream tube. This is a viscous effect, that is, the flow in a stream tube, which is moving faster than that in an adjacent stream tube does work on the slower-moving fluid. Note that this is accompanied by a continual generation of waste heat, that is, irreversible entropy production, because the receiving (slower flow) stream tube sees less actual work then that given up by the faster moving stream tube. The balance is realized as internal heat generation.

The energy equation for this stream tube (assuming no net shaft work interaction with the surroundings) is written as follows:

$$\dot{H}_{\text{te}} = \dot{H}_{\text{ti}} + \dot{Q} + \int_i^e \mathrm{d}\boldsymbol{F}_{\text{bound(shear)}} \cdot \boldsymbol{V}_{\text{bound}} \tag{1}$$

Here $\dot{H}_t$ is the total enthalpy flow rate across a given cross section of the streamtube and is a function of $\dot{m}$, T, P, species composition, that is, α_l (species mass fractions.) and $\boldsymbol{V}$.

The mass flow rate equation (continuity) is given by

$$\dot{m}_e = \dot{m}_i \qquad \text{with} \quad \dot{m}_e = \rho_e|\boldsymbol{V}_e|A_e \qquad \text{and} \qquad \dot{m}_i = \rho_i|\boldsymbol{V}_i|A_i \tag{2}$$

The second law for the stream tube can be written in terms of entropy flow rate as follows:

$$\dot{S}_e = \dot{S}_i + \dot{S}_{\text{irr(internal)}} + \dot{m}\int_i^e \frac{\delta q}{T} \tag{3}$$

Here $\dot{S}_{\text{irr(internal)}}$ is the rate of the irreversible generation of entropy because of internal irreversibilites (friction, etc.) occurring in the stream tube. The heat term is the production of entropy caused by heat transferred from the boundary into the stream tube. This term can be separated into a reversible entropy increase (i.e., the entropy increase that would be experienced by the fluid for the given heat interaction if that interaction occurred reversibly at the total temperature of the fluid T_t) and an irreversible entropy increase associated with the losses of the (actual) heat interaction caused by the static temperature being less than the total temperature. This is written as follows:

$$\dot{S}_e = \dot{S}_i + \dot{S}_{\text{irr(internal)}} + \dot{S}_{\text{irr(heat transfer)}} + \dot{S}_{\text{rev}} = \dot{S}_e = \dot{S}_i + \dot{S}_{\text{irr}} + \dot{S}_{\text{rev}} \tag{4}$$

Here $\dot{S}_{\text{rev}} = \dot{m}\int(\delta q/T_t)$ and the entropy flow rate $\dot{S}$ is a function of the following: $\dot{m}$, T, P, and species composition, that is, α_l (species mass fractions).

The gas equation of state at the exit is

$$P_e = \rho_e R_e T_e \tag{5}$$

where R_e is the gas constant (function of the species).

The velocity vectors at stream-tube entrance and stream-tube exit can be defined in terms of their components, that is,

$$\boldsymbol{V}_i = u_i\hat{i} + v_i\hat{j} + w_i\hat{k}$$
$$\boldsymbol{V}_e = u_e\hat{i} + v_e\hat{j} + w_e\hat{k} = |\boldsymbol{V}_e|(\hat{n}_{\text{xe}}\hat{i} + \hat{n}_{\text{ye}}\hat{j} + \hat{n}_{\text{ze}}\hat{k}) \tag{6}$$

The unit vector components $\hat{n}_{\text{xe}}$, $\hat{n}_{\text{ye}}$, $\hat{n}_{\text{ze}}$ define the angularity of the exiting flow.

Equations (1), (2), (4), and (5) demonstrate that for known 1) stream-tube orientation (angularity of entering and exiting flow); 2) irreversible entropy generation in the stream tube, 3) given energy interactions, boundary viscous work, and chemical composition and 4) given inflow conditions and stream-tube areas, the exit plane fluid dynamics u_e, v_e, w_e, P_e, T_e, and ρ_e are uniquely determined (or calculable) from this system of equations.

Now, utilizing the momentum equation, the following expressions for the axial, vertical, and side forces on the fluid in the stream tube (from side boundaries) can be written:

$$F_x = \rho_e|\boldsymbol{V}_e|u_e A_e + P_e A_e\hat{n}_{\text{xe}} - \rho_i|\boldsymbol{V}_i|u_i A_i - P_i A_i\hat{n}_{\text{xi}} \tag{7}$$

$$F_y = \rho_e|\boldsymbol{V}_e|v_e A_e + P_e A_e\hat{n}_{\text{ye}} - \rho_i|\boldsymbol{V}_i|v_i A_i - P_i A_i\hat{n}_{\text{yi}} \tag{8}$$

$$F_z = \rho_e|\boldsymbol{V}_e|w_e A_e + P_e A_e\hat{n}_{\text{ze}} - \rho_i|\boldsymbol{V}_i|w_i A_i - P_i A_i\hat{n}_{\text{wi}} \tag{9}$$

Here, $\boldsymbol{F}_{\text{streamtube}} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$.

Because the force components are functions of the exit flow conditions, the functional dependence of the force components on the stream tube can be written as

$$F_x = \text{function(inflow conditions, } \dot{Q},\ A_e/A_i, \text{ exit flow angularity, } \dot{S}_{\text{irr}})$$
$$F_y = \text{function(inflow conditions, } \dot{Q},\ A_e/A_i, \text{ exit flow angularity, } \dot{S}_{\text{irr}})$$
$$F_x = \text{function(inflow conditions, } \dot{Q},\ A_e/A_i, \text{ exit flow angularity, } \dot{S}_{\text{irr}}) \tag{10}$$

or

$$\boldsymbol{F}_{\text{streamtube}} = \text{function(inflow conditions, } \dot{Q},\ A_e/A_i, \text{ exit flow angularity, } \dot{S}_{\text{irr}})$$

Here, *function* refers merely to a general but useful descriptor of the functional dependence for the force or force components on the various terms in the parentheses. Inflow conditions refer to stream-tube entrance pressure, velocity, temperature, area, etc. For simplicity in Eq. (10) and in the following discussion, the exit plane species composition is not shown as a functional driver, that is, constant composition in the stream tube is assumed with a given gas constant, etc. Note however that the methodology is general and can easily include variable species composition effects.

Also for a single stream tube as shown, the functional dependence for the forces would properly include the already discussed shear boundary work term

$$\int_i^e \mathrm{d}\boldsymbol{F}_{\text{bound(shear)}} \cdot \boldsymbol{V}_{\text{bound}}$$

This term, however, is not shown in Eq. (10) because for a collection of stream tubes that are "bundled" together (as over an aerospace vehicle which is of interest here) but that are at different conditions, and when force effects are summed/integrated over the entire vehicle control volume of interest (as will be done in the next section), the shear boundary work term effectively cancels out between adjacent internal stream tubes with the net effect of the viscosity appearing as an increase in entropy.

In examining the functional dependence for stream-tube forces, it is seen that if one is interested *solely* in the impact of irreversibility it is necessary to evaluate the reversible forces with the other terms in the functional dependence held constant, that is, define reversible force components $F_{x(\text{rev})}$, $F_{y(\text{rev})}$, $F_{z(\text{rev})}$ as

$$F_{x(\text{rev})} = \text{function(inflow conditions, } \dot{Q},\ A_e/A_i,$$
$$\text{exit flow angularity}, \dot{S}_{\text{irr}} = 0)$$
$$F_{y(\text{rev})} = \text{function(inflow conditions, } \dot{Q},\ A_e/A_i,$$
$$\text{exit flow angularity}, \dot{S}_{\text{irr}} = 0)$$
$$F_{x(\text{rev})} = \text{function(inflow conditions, } \dot{Q},\ A_e/A_i,$$
$$\text{exit flow angularity}, \dot{S}_{\text{irr}} = 0) \tag{11}$$

or

$$\boldsymbol{F}_{\text{rev}-\text{streamtube}} = F_{x(\text{rev})}\hat{i} + F_{y(\text{rev})}\hat{j} + F_{z(\text{rev})}\hat{k}$$
$$= \text{function(inflow conditions, } \dot{Q},\ A_e/A_i,$$
$$\text{exit flow angularity}, \dot{S}_{\text{irr}} = 0)$$

One can similarly define the isentropic (reversible and adiabatic) force and force components on the fluid in the stream tube as follows:

$$F_{x(\text{isen})} = \text{function(inflow conditions, } \dot{Q} = 0,\ A_e/A_i,$$
$$\text{exit flow angularity}, \dot{S}_{\text{irr}} = 0)$$
$$F_{y(\text{isen})} = \text{function(inflow conditions, } \dot{Q} = 0,\ A_e/A_i,$$
$$\text{exit flow angularity}, \dot{S}_{\text{irr}} = 0)$$
$$F_{x(\text{isen})} = \text{function(inflow conditions, } \dot{Q} = 0,\ A_e/A_i,$$
$$\text{exit flow angularity}, \dot{S}_{\text{irr}} = 0) \tag{12}$$

or

$$\boldsymbol{F}_{\text{isen}-\text{streamtube}} = F_{x(\text{isen})}\hat{i} + F_{y(\text{isen})}\hat{j} + F_{z(\text{isen})}\hat{k}$$
$$= \text{function(inflow conditions, } \dot{Q} = 0,\ A_e/A_i,$$
$$\text{exit flow angularity}, \dot{S}_{\text{irr}} = 0)$$

The force lost caused by irreversibility within the stream tube can then be defined as follows:

$$\Delta\boldsymbol{F}_{\text{irr}-\text{streamtube}} = \boldsymbol{F}_{\text{rev}-\text{streamtube}} - \boldsymbol{F}_{\text{streamtube}} \tag{13}$$

The force increment caused by heat interaction within the stream tube can also be defined as

$$\Delta\boldsymbol{F}_{\text{heat}-\text{streamtube}} = \boldsymbol{F}_{\text{rev}-\text{streamtube}} - \boldsymbol{F}_{\text{isen}-\text{streamtube}} \tag{14}$$

This then leads to the *decomposition* of the actual force felt by the fluid in the stream tube as

$$\boldsymbol{F}_{\text{streamtube}} = \boldsymbol{F}_{\text{isen}-\text{streamtube}} - \Delta\boldsymbol{F}_{\text{irr}-\text{streamtube}} + \Delta\boldsymbol{F}_{\text{heat}-\text{streamtube}} \tag{15}$$

Based on Equations (10–12) and the preceding discussion, it is relatively straightforward to construct two sequential solvers that compute both $\boldsymbol{F}_{\text{rev}-\text{streamtube}}$ and $\boldsymbol{F}_{\text{isen}-\text{streamtube}}$ by processing the exit plane fluid dynamics of the stream tube. These two processes are briefly described for a perfect gas in the following discussion.

A. Solver 1: Computation of Reversible Forces by Removal of Irreversibilites at Stream-Tube Exit

This entails a separate correction at the exit plane of the stream tube based on the solution of the following equations for the reversible exit conditions:

$$\dot{m}_e = \rho_{e,r} V_{e,r} \cos\beta A_e \tag{16}$$

$$C_P T_{e,r} + V_{e,r}^2/2 = C_P T_e + V_e^2/2 \tag{17}$$

$$s_{\text{irr(streamtube)}} = C_P\ \ell\text{n}(T_e/T_{e,r}) - R\ \ell\text{n}(P_e/P_{e,r}) \tag{18}$$

$$P_{e,r} = \rho_{e,r} R T_{e,r} \tag{19}$$

β is the actual turning angle of the flow (known), and $s_{\text{irr(streamtube)}}[= \dot{S}_{\text{irr(streamtube)}}/\dot{m}]$ must be known from the fluid dynamics of the actual flowfield (calculated or known for the given stream tube). Note that s_{irr} is only the irreversible entropy increase per mass—it does not include any entropy increase associated with reversible heat addition from across the boundary of a stream tube.

The preceding system must be solved for $V_{e,r}$, $T_{e,r}$, $P_{e,r}$, and $\rho_{e,r}$. These "reversible" exit plane fluid dynamics can then be directly used to compute $\boldsymbol{F}_{\text{rev}-\text{streamtube}}$ and its x, y, and z components. These force components exactly represent the forces that would be experienced by the fluid in the stream tube if the flowfield were entirely reversible with all other drivers held constant.

B. Solver 2: Removal of Heat Added to Stream Tube

A second solver, which processes the reversible exit plane (obtained in the preceding first solver), can then be constructed. The purpose of this solver is to compute the fluid dynamics at the exit plane when any heat added across the boundary of the stream tube (from freestream to exit plane as it was processed around or through the vehicle) is removed. Keep in mind that this solver works on the reversible exit plane obtained from the preceding first solver. Subsequently the force $\boldsymbol{F}_{\text{isen}-\text{streamtube}}$ can be calculated; this force vector is in fact exactly the force vector on the stream tube caused simply by isentropic expansion from freestream to exit area and (isentropic) flow turning of the stream tube.

This second solver can be constructed as follows:

$$\dot{m}_e = \rho_{\text{e,isen}} V_{\text{e,isen}} \cos\beta A_e \tag{20}$$

$$C_P T_{\text{e,isen}} + V_{\text{e,isen}}^2/2 = C_P T_{e,r} + V_{e,r}^2/2 - (\dot{Q}/\dot{m}) \tag{21}$$

$$C_P\ \ell\text{n}(T_{e,r}/T_{\text{e,isen}}) - R\ \ell\text{n}(P_{e,r}/P_{\text{e,isen}}) = s_{\text{rev}} \tag{22}$$

$$P_{\text{e,isen}} = \rho_{\text{e,isen}} R T_{\text{e,isen}} \tag{23}$$

In this solver $q(=\dot{Q}/\dot{m})$ and s_{rev} (per unit mass) are known from the actual flowfield for the given stream tube. This system must be solved for $V_{e,\text{isen}}$, $P_{e,\text{isen}}$, $T_{e,\text{isen}}$, and $\rho_{e,\text{isen}}$, corresponding to conditions at the end of the given stream tube for upstream isentropic (reversible and no heat interaction) conditions. These isentropic exit plane fluid dynamics can then be directly used to compute $\boldsymbol{F}_{\text{isen}-\text{streamtube}}$ and its x, y, and z components. These force components exactly represent the forces that would be experienced by the fluid in the stream tube if the flowfield was entirely isentropic.

Therefore, using this methodology [with Eqs. (13) and (14)], the actual force components on the stream tube (i.e., $F_{x,\text{streamtube}}$, $F_{y,\text{streamtube}}$, and $F_{z,\text{streamtube}}$) can be "deconstructed" as the additive summation of the force increments caused by irreversibility, heat addition, and isentropic area change and (isentropic) flow turning.

III. Vehicle Control Volume Selection and Force Decomposition

This section provides the general methodology for the methodical deconstruction and resulting audit of high-speed vehicle performance in terms of irreversibilities occurring in the flow, heat added to the flow, and area relief and flow turning inherent in stream-tube processing around and through the vehicle. The discussion in the preceding section, which focused on individual stream tubes, is now expanded to the overall control volume (or global stream tube) encompassing an entire aerospace vehicle. However the individual stream-tube concepts discussed earlier are utilized in analyzing the specifics of the flowfield (and forces) within that overall control volume.

To accomplish this, first construct a comprehensive fluid control volume around a hypersonic vehicle in flight as shown below in Fig. 2 such that the box defining this control volume extends on either side to freestream conditions, that is, the zone of influence of the vehicle impacts only the downstream boundary of the control volume as shown (i.e., the vehicle exit plane). The vehicle solid structure is not part of the control volume, which simply encompasses all fluids internal and external to the vehicle. Therefore, although not shown on the sketch, this control volume can have internal flow paths within the structure of the vehicle, including fuel/propellant lines and systems extending all of the way back into and including the volumes of the propellant tanks themselves. This indicates that vehicle wetted surfaces refer to all wetted surfaces of the solid structure of the vehicle, internal and external. For simplicity assume that the fluid dynamics everywhere where there is fluid motion is steady (although propellant is of course being emptied out of propellant tanks). Formally this assumption mandates steady cruise although the analysis will hold generally unless there is extremely rapid acceleration/deceleration of the vehicle such that the fluid dynamics over the vehicle or through the engine flow path are unsteady. The net result of all pressure and shear forces acting from the fluid on all wetted surfaces of the vehicle is a resultant overall fluid dynamic force vector $\boldsymbol{F}_{\text{vehicle}}$, which is further designated in terms of x, y, and z components $F_{x(\text{veh})}$, $F_{y(\text{veh})}$, and $F_{z(\text{veh})}$. Note that $F_{x(\text{veh})}$ can be related to conventional thrust − drag, whereas $F_{y(\text{veh})}$ and $F_{z(\text{veh})}$ can similarly be thought of as conventional lift force and side force, respectively, but these distinctions are fundamentally arbitrary and will not be used at this time.

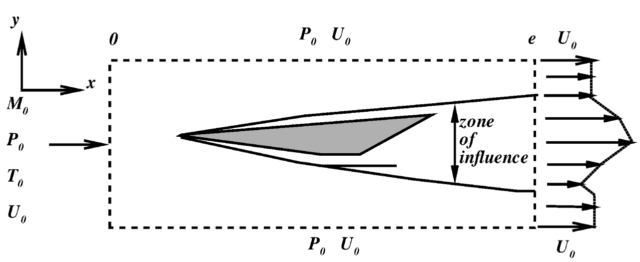

Fig. 2 Comprehensive vehicle control volume (includes propulsion system and fuel system).

For this vehicle and with the given assumptions, 1) the overall (actual) force vector on the vehicle wetted surfaces (internal and external) $\boldsymbol{F}_{\text{vehicle}}$, 2) the overall reversible vehicle force vector $\boldsymbol{F}_{\text{rev-vehicle}}$, and 3) the overall isentropic vehicle force vector $\boldsymbol{F}_{\text{isen-vehicle}}$ can be written as follows:

$$\boldsymbol{F}_{\text{vehicle}} = \sum_{j=1}^{N} \boldsymbol{F}_{\text{streamtube}}, \qquad \boldsymbol{F}_{\text{rev-vehicle}} = \sum_{j=1}^{N} \boldsymbol{F}_{\text{rev-streamtube}}$$
$$\boldsymbol{F}_{\text{isen-vehicle}} = \sum_{j=1}^{N} \boldsymbol{F}_{\text{isen-streamtube}} \tag{24}$$

Here N is the total number of stream tubes modeled within the domain of influence of the vehicle. Fluid forces between adjacent surfaces of stream tubes cancel out such that all that is left is the net force on the vehicle wetted surfaces.

Thus, for the entire vehicle, consistent with the earlier discussion,

$$\boldsymbol{F}_{\text{vehicle}} = \sum_{i=1}^{N} (\boldsymbol{F}_{\text{isen-streamtube}} + \Delta\boldsymbol{F}_{\text{heat-streamtube}} - \Delta\boldsymbol{F}_{\text{irr-streamtube}}) \tag{25}$$

This implies that one can define the overall lost force caused by irreversibility for the entire vehicle as

$$\Delta\boldsymbol{F}_{\text{irr(vehicle)}} = \sum_{i=1}^{N} \Delta\boldsymbol{F}_{\text{irr-streamtube}} \tag{26}$$

Also, the overall force contribution as a result of heat interaction for the vehicle is

$$\Delta\boldsymbol{F}_{\text{heat(vehicle)}} = \sum_{i=1}^{N} \Delta\boldsymbol{F}_{\text{heat-streamtube}} \tag{27}$$

This analysis corresponds to the systematic deconstruction of vehicle forces as illustrated in Fig. 3: Solver 1 applied to the exit

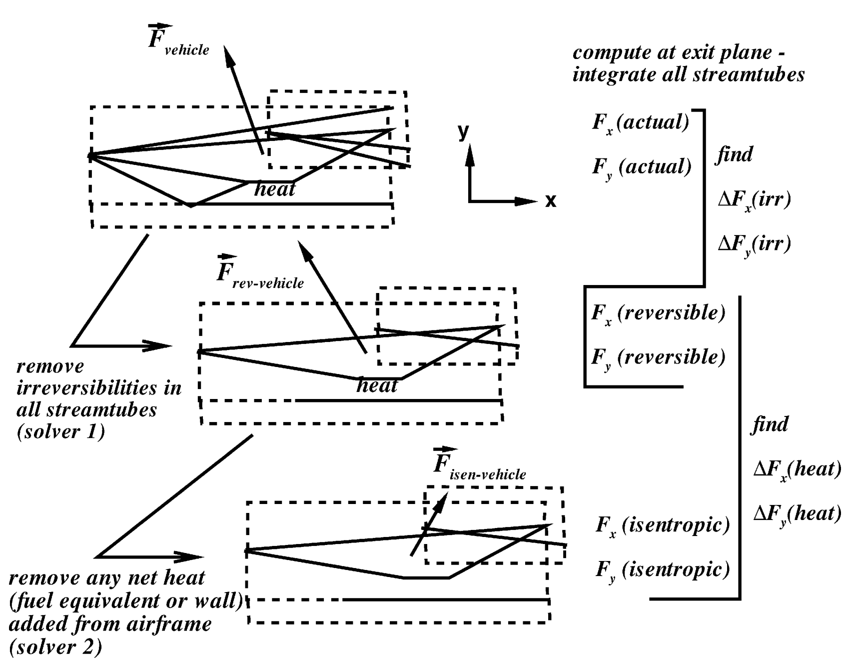

Fig. 3 Vehicle force audit (decomposition) process.

planes of all modeled stream tubes within the vehicle domain of influence out to the exit plane of the vehicle provides the reversible performance $F_{x(\text{rev-vehicle})}$, $F_{y(\text{rev-vehicle})}$, and $F_{z(\text{rev-vehicle})}$ of the vehicle with given energy interactions $\dot{Q}$ across the boundaries of the overall control volume. Solver 2, applied to the exit planes of the reversible stream tubes as determined by solver 1, then provides the overall performance increment of the vehicle caused by the heat interaction. The base force components $F_{x(\text{isen-vehicle})}$, $F_{y(\text{isen-vehicle})}$, and $F_{z(\text{isen-vehicle})}$ are then the forces that exist simply because of the net effect of stream-tube area changes and flow turning over the vehicle.

IV. Simplified High-Speed Example Using Force Auditing Methodology

Consider a highly simplified two-dimensional hypersonic vehicle as shown in Fig. 4a. For this example, the flight Mach number is 12, and ambient temperature and pressure are 200 K and 1000 N/m^2, respectively. The air is calorically perfect with constant specific heats throughout the flow. The freestream capture area is 1 m^2, and the tip to tail length of the vehicle is 10.2 m. The inlet has an incident and a reflected shock from cowl tip as shown in the Fig. 4a, which is generated by an 8-deg forebody compression angle; with reflected shock cancellation at combustor shoulder. Heat per unit mass of air is added in the (Rayleigh) combustor such that the Mach number at combustor exit (nozzle entrance) is Mach 2. (The heat rate added to the flow path in the combustor is 1.75e8 W.) The flow in the propulsion system from combustor entrance to nozzle exit is quasi one dimensional. The top surface of the vehicle has an 8-deg turning angle such that an oblique shock is generated. A flat-plate wing is separately defined, at 8-deg positive angle of attack, and with the same planform area as the top surface of the vehicle; the wing has no interaction with the body. The uniformity of angles and areas for the various components are selected purely for convenience and easy repeatability for this particular example.

The problem as defined is inviscid everywhere; the only irreversibilities in this problem are shock (inlet, top surface, wing) and Rayleigh losses in the combustor. It should be emphasized that the methodology given in preceding sections is very general and is formally independent of the level of analysis used to actually generate the flowfield. Of course, the fidelity of the results obtained using the methodology will be the same as the fidelity of the original modeling used to generate the flowfield. The example shown in this work is constrained to be highly conceptual for simplicity and is defined merely to demonstrate the technique described earlier. However, the methodology has been applied successfully to far more complex (and realistic) vehicle configurations with friction, fuel injection, reaction, etc. These results are beyond the scope of the current paper.

The flowfield around the simplified vehicle shown here is divided into five relevant stream tubes for analysis; an upper or top surface stream tube, an engine flowpath stream tube, a cowl-side (lower) stream tube, a shocked (lower) wing stream tube, and an upper (already reversible) expansion-side stream tube on the wing (see Fig. 4b).

The results obtained by applying this methodology are shown in Figs. 5 and 6, which detail the force audit in terms of axial x and vertical y components. At the level of modeling used here and with the simplifying assumptions made to set this problem up, the only stream tubes contributing to the y direction are the wing and upper surface stream tubes (or control volumes), that is, the engine flowpath stream tube and the cowl-side stream tube are axially directed and undergo no net change in momentum in the y direction.

Note in Fig. 5 that the resolved axial force on the vehicle in the absence of all irreversibility and heat addition (13,489 N) is

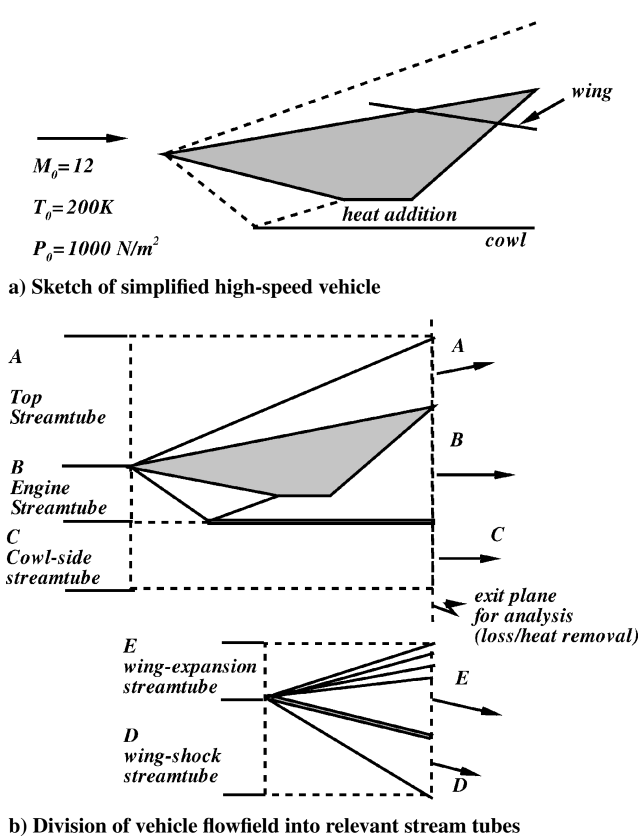

Fig. 4 Simplified vehicle for methodology demonstration.

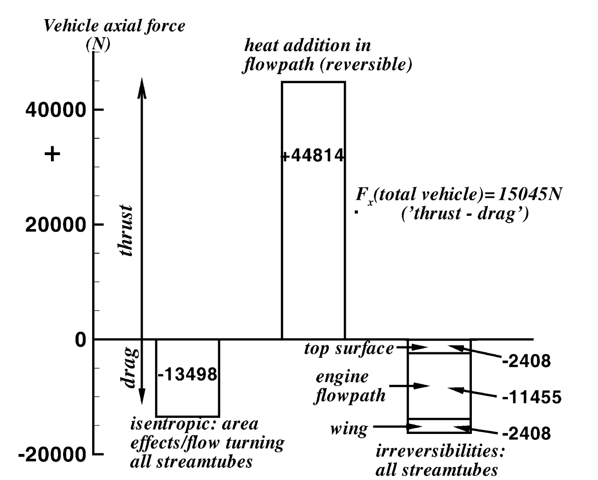

Fig. 5 Axial-force audit for simplified hypersonic vehicle problem.

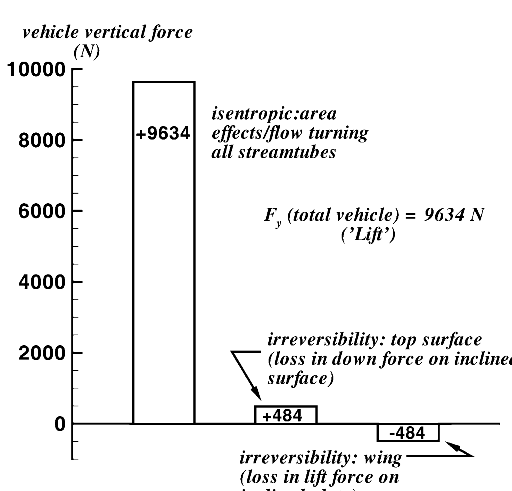

Fig. 6 Vertical-force audit for simplified hypersonic vehicle example.

negative, that is, is a drag contribution. The lost axial force caused by irreversibilities sum to 16,271 N (total) but are shown subdivided by relevant control volume (stream tube) with the propulsion flowpath representing the largest loss. Finally, the net (overall) gain of 15,045 N in the axial direction for the entire vehicle is caused by the large increment associated with heat addition in the combustor of the engine flowpath stream tube (45,000 N).

Figure 6 shows the vertical force audit for this example. The irreversibilities associated with the upper surface cause a reduction in the downforce on that surface; similarly the irreversibilities associated with the wing (which simply for ease of constructing this problem had the same angle of attack as the upward angle of inclination of the top surface and the same surface area as well) cause a similar reduction in the upward force on that surface. Hence these lost force increments in the vertical direction on these two opposing surfaces cancel out, as they should. Obviously, in general this will not be so but happens here because of the particular details of the simplified modeling of the configuration.

V. Relationship of Wake Irreversibility to Vehicle Performance—Theoretical Development

The remainder of this paper provides additional analysis and theoretical framework, which directly relates the wake-mixing process that occurs downstream of an aerospace vehicle to the developed force(s) experienced by that vehicle. Specifically, the entropy production associated with that wake-mixing process is shown to be the key component for the (entropy-based) calculation of vehicle force. Hence, overall entropy production (over and through the vehicle as well as in the wake) is directly related to vehicle axial-force development; furthermore, in the same development conventional availability is also related directly to vehicle axial-force development. Optimal performance for given energy interactions from and to the vehicle is therefore shown (analytically) to occur when total irreversibility is minimized across both vehicle flowfield and in the (downstream) wake.

A. Discussion of the Relationships Between Overall Vehicle Axial Force, Vehicle Exit Plane, and the Wake-Mixing Process

The flow at the exit plane of the aerospace vehicle (see Fig. 2) is characterized by a wide variety of flow conditions and flow angularity (distortion) corresponding to the multiple stream tubes within the vehicle-bounded flow. Furthermore, in the wake all stream tubes [both external streamtubes that wrap around the vehicle upstream as well as the streamtube(s), which were processed in internal (engine) flowpaths] mix to uniformity (i.e., equilibrium) with the freestream. In this wake-mixing process, there is a complete loss of work potential (i.e., maximum entropy production caused by internal irreversibilities within the mixing process—no recovery of force-based work). The maximum thermodynamically permissible limit would be isentropic (overall) mixing; however, in order to obtain the required internal reversible processes force-based interactions with the control system boundary are required. This is not allowable, of course, in the unconstrained vehicle wake. The net effect of the vehicle out to its own exit plane is then to separate the freestream into various stream tubes and process them in various ways. In so doing, actual forces $F_{x(\mathrm{veh})}$ and $F_{y(\mathrm{veh})}$ result on the vehicle as analyzed in preceding sections, inevitably with the generation of irreversibilities within the flowpath(s). It is now of interest, however, to consider the relationship of the vehicle wake to achieved vehicle performance.

B. Theoretical Framework for Relating Wake-Mixing Processes to Vehicle Forces Using the Laws of Thermodynamics

Consider a high-speed vehicle embedded within an encompassing global stream tube as shown in Fig. 7 (for which the far-field side boundaries are aligned with the x axis). The global stream tube is initially processed from i to e over the vehicle and is inclusive of both the vehicle zone of influence control volumes discussed earlier and side control volumes above and below the vehicle, which are entirely in the freestream. In addition, as shown in Fig. 7, the wake control volume is constructed behind the vehicle as shown and extends from e to w, as shown. By definition, there is no net force

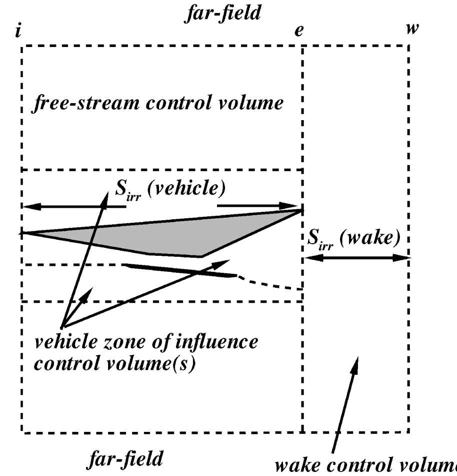

Fig. 7 Global control volume/stream-tube definition for overall vehicle/wake analysis.

on the wake control volume; hence, there is no contribution from it to vehicle forces. It extends from far field below to far field above; the side control volumes in the upstream freestream patch directly into this wake control volume. The question of how large to make this control volume in the y direction is not of great interest at this time—it must however be large enough to encompass the wake mixing. The actual length of this wake zone is also not of interest from the standpoint of the vehicle performance as will be shown next. The only important information from the standpoint of the wake will be the irreversibility associated with the wake-mixing process as the multiple nonuniform stream tubes at vehicle exit plane e mix out to a condition of displaced equilibrium at w. The global stream tube by definition has constant cross-sectional area, that is, $A_i = A_e = A_w$. Initially (in this section), assume perfect gas (nonreacting) and furthermore that inflow i and exit w cross-sectional planes of the stream tube are aligned with the y axis (i.e., the directed unit normal is entirely in the axial direction on both inflow and outflow planes). It is relatively simple to write the following important relationship for this stream tube:

$$\dot{Q} + \dot{W} = \dot{m}C_P(T_w - T_i) - u_i(P_w A_w - P_i A_i) + \dot{m}\left[(u_w - u_i)^2\big/2\right] + u_i F_{x(\mathrm{veh})} \tag{28}$$

Hence $F_{x(\mathrm{veh})}$ is essentially the familiar resultant force of thrust − drag while $\dot{Q}$ and $\dot{W}$ are the net heat and work rate interactions crossing the boundary into the fluid from the structure of the vehicle (positive to the fluid). For a ramjet/scramjet-powered vehicle the work term is obviously zero; however, even for conventional jet engines with turbomachinery the net work is close to zero. Note also that the last term $u_i F_{x(\mathrm{veh})}$ is by simple extension the conventional (thrust-drag power) for a high-speed vehicle, and the energy interactions (heat + work) are the price paid by the vehicle.

The wake process mixes (with no recovery of work potential) both the vehicle exit plane fluid dynamics and the freestream to some condition just displaced from the freestream condition (assuming the side boundaries of the global stream-tube/wake-mixing streamtube are taken well into the far field). In other words, the following can be written:

$$u_w = u_i + \Delta u, \qquad T_w = T_i + \Delta T, \qquad P_w = P_i + \Delta P \tag{29}$$

etc., where Δu, ΔT, ΔP are very small.

Equation (28) can then be applied across the global stream tube from inflow i to wake exit w such that the following can be written:

$$\dot{Q} + \dot{W} = \dot{m}C_P \Delta T - \dot{m}(\Delta P/\rho_i) + \dot{m}[(\Delta u)^2/2] + u_i F_{x(\mathrm{veh})} \tag{30}$$

From the definition of the total entropy change from i to w (and discarding higher order terms), the following is then written:

$$\dot{Q} + \dot{W} = T_i \dot{S}_{\mathrm{total}} + u_i F_{x(\mathrm{veh})} \tag{31}$$

Vehicle axial-force power is then

$$u_i F_{x(\text{veh})} = \dot{Q} + \dot{W} - T_i \dot{S}_{\text{total}} \tag{32}$$

where

$$\dot{S}_{\text{total}} = \dot{S}_{\text{vehicle}} + \dot{S}_{\text{wake}} = \dot{S}_{\text{irr(vehicle)}} + \dot{S}_{\text{rev(vehicle)}} + \dot{S}_{\text{wake}} \tag{33}$$

Hence $\dot{S}_{\text{total}}$ is the total entropy change caused by 1) irreversibilities occurring over the vehicle and energy transfer in the form of heat transferred from the vehicle to the flow, that is, $\dot{S}_{\text{vehicle}}$ and critically 2) all irreversibility occurring in the wake, that is, $\dot{S}_{\text{wake}}$. Maximum $F_{x(\text{veh})}$ (i.e., thrust − drag, where x is in direction of freestream approach to the vehicle) occurs when the sum of the irreversible entropy generation rates for the vehicle zone of influence (out to the exit plane of the vehicle) and the wake process is minimized (for given energy interactions with the vehicle):

$$MAX\ F_{x(\text{veh})} \rightarrow \left\{\dot{S}_{\text{irr(vehicle)}} + \dot{S}_{\text{irr(wake)}}\right\}_{\text{MIN}} \tag{34}$$

Note that this clarifies the issue of isentropic drag: one can easily construct an aerodynamic free-flying closed shape with isentropic flow everywhere that nevertheless has net pressure drag acting on it. The drag is a penalty (not even necessarily with any irreversibilities occurring in the flow over the shape) caused by flow turning out of the axial direction and the separation of stream tubes around the shape. In a very real sense, then, the out-of-equilibrium situation at the exit plane of the aerodynamic shape and the inevitable downstream loss in work potential in the wake equilibration process provides a measure (via the irreversibility generated in that wake process) of the drag experienced by the shape. Irreversibilities in the actual flow over the shape will of course increase the actual drag above this isentropic flow + lossy wake mixing limit.

Furthermore, one can also define the lost axial-force component (reversible axial force—actual axial force) for a hypersonic aerospace vehicle as follows:

$$\Delta F_{x(\text{irr-veh})} = F_{x(\text{rev-veh})} - F_{x(\text{veh})}$$

$$= T_i\left[\dot{S}_{\text{irr(vehicle)}} + \dot{S}_{\text{wake}} + \dot{S}_{\text{wake(reversible vehicle)}}\right]\Big/u_i \tag{35}$$

Note that $\dot{S}_{\text{irr(vehicle)}}$ is the entropy generation associated with irreversibility out to the exit plane of the vehicle e, that is, it does not include any entropy increment caused by reversible heat interaction [i.e., it excludes

$$\dot{S}_{\text{rev(vehicle)}} = \int_i^e \frac{\delta q}{T_t}$$

where T_T is the local total temperature of the fluid].

$\dot{S}_{\text{wake}}$ is the entropy generation rate in the wake-mixing process for the actual flow while $\dot{S}_{\text{wake(reversible vehicle)}}$ is the entropy generation rate in the wake-mixing process for the flow with all processes reversible as defined in earlier sections.

C. Important Note on Relating Axial Forces to Entropy Production

It must be cautioned that it is tempting to simply take Eq. (32) and directly compute various subdivided lost axial-force increments based on the local entropy rates (as is done in conventional exergy analysis); if this is done, there is then no distinction in terms of the upstream fluids—all heat/work/irreversibility are "created equal." However, this would ignore the critical fact that different stream tubes processed over and through the vehicle have different force characteristics depending on their own history/evolution out to the exit plane of the vehicle. It must be considered that there is significant and inevitable irreversibility occurring in the wake process [although treated as a lumped process in Eq. (32)], which should itself be associated with individual upstream stream tubes (in terms of flow turning, angularity, area, total conditions, etc.). This leads back to the performance analysis methodology in terms of control volume/stream-tube analysis developed in the earlier sections of this paper; the theoretical framework presented here does not change the issues or results involved with the detailed force audit methodology as presented earlier.

VI. Wake-Mixing Process

The quantification of the entropy associated with the wake mixing $\dot{S}_{\text{wake}}$ is easily realized by the use of an adiabatic inviscid-walled mixing (or one-dimensionalization) process defined across a chosen control volume in the wake and encompassing the freestream out to the far field (see Fig. 7).

This process is sometimes called stream-thrust averaging and is well known for obtaining a one-dimensional representation of a multidimensional flowfield. This technique simply equates one-dimensionalized mass flow rate, one-dimensionalized stream thrusts, and one-dimensionalized total enthalpy flow rate to the actual (integrated) three-dimensional mass flow rate, stream thrusts, and total enthalpy flow rate. The system is closed using the gas equation of state. For multispecies flows, the one-dimensionalized mass fractions α_l are readily found from the integrated species flow rates as well. This allows the calculation of the one-dimensional gas constant. In summary, the following system of equations is solved for one-dimensional unknowns u_w, v_w, w_w, P_w, T_w, and ρ_w:

$$\int_{A_e} \rho(\boldsymbol{V}\cdot\hat{n})\,\mathrm{d}A = \rho_w u_w A_w \tag{36}$$

$$\int_{A_e} [\rho u(\boldsymbol{V}\cdot\hat{n}) + P\hat{n}_x]\,\mathrm{d}A = \left(\rho_w u_w^2 + P_w\right)A_w \tag{37}$$

$$\int_{A_e} [\rho v(\boldsymbol{V}\cdot\hat{n}) + P\hat{n}_y]\,\mathrm{d}A = \rho_w u_w v_w A_w \tag{38}$$

$$\int_{A_e} [\rho w(\boldsymbol{V}\cdot\hat{n}) + P\hat{n}_y]\,\mathrm{d}A = \rho_w u_w w_w A_w \tag{39}$$

$$\int_{A_e} \rho h_t(\boldsymbol{V}\cdot\hat{n})\,\mathrm{d}A = \rho_w h_{tw} u_w A_w \tag{40}$$

and

$$P_w = \rho_w R_w T_w \tag{41}$$

where

$$h_t = \sum_{\ell=1}^{NCS}\alpha_\ell\left(h_{t_{0,\ell}} + \int_{T_{\text{ref}}}^{T} C_{p,\ell}\,\mathrm{d}T\right) + \frac{u^2}{2} + \frac{v^2}{2} + \frac{w^2}{2} \tag{42}$$

$$h_{t,w} = \sum_{\ell=1}^{NCS}\alpha_{\ell,w}\left(h_{t_{0\ell}} + \int_{T_{\text{ref}}}^{T_0} C_{p\ell}\,\mathrm{d}T\right) + \frac{u_w^2}{2} + \frac{v_w^2}{2} + \frac{w_w^2}{2} \tag{43}$$

and

$$R_w = R_{\text{univ}}/mw_{\text{tot,w}} \tag{44}$$

and

$$mw_{\text{tot,w}} = 1\Bigg/\sum_{\ell=1}^{NCS}\frac{\alpha_{\ell,w}}{mw_\ell} \tag{45}$$

In practice, when the wake control volume is defined, the size of the side control volume associated with the freestream (from outer edge of the vehicle zone of influence to the far field) is somewhat arbitrary but should be made as large as permissible from the standpoint of machine capabilities to compute accurately (i.e., for the example to be shown in the next section, the ratio of overall global stream-tube cross-sectional area to vehicle exit plane area was varied to extremely large values—up to 50,000 with asymptotic convergence exhibited however at much smaller ratios).

The entropy production associated with this wake process can then be directly computed as the difference in exiting to entering (vehicle exit plane) entropy flow rate, that is,

$$\dot{S}_{\text{wake}} = \rho_w u_w s_w A_w - \int_{A_e} \rho(\boldsymbol{V}\cdot\hat{n})s\,\mathrm{d}A_i \tag{46}$$

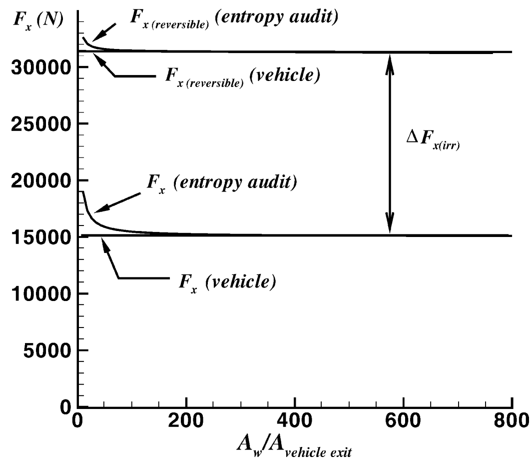

Fig. 8 Axial force on simplified hypersonic vehicle and axial force computed using entropy audit [Eq. (32)] vs size of global stream tube.

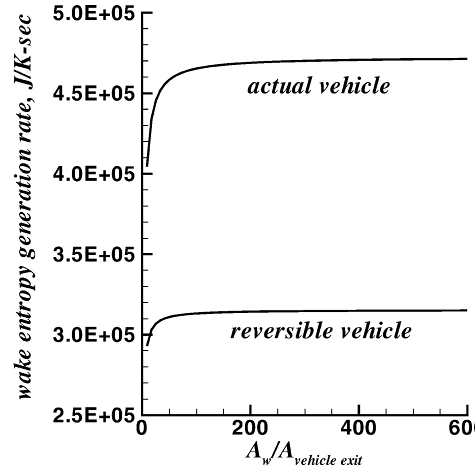

Fig. 9 Wake entropy generation rates for simplified vehicle vs size of global stream tube.

VII. Generic High-Speed Example—Wake Mixing and Axial Force on Vehicle

Consider the generic simplified high-speed vehicle depicted in Fig. 4 and described earlier. Figure 8 shows the results of applying Eq. (32) to the fluid dynamics associated with the simplified high-speed vehicle. Specifically, in this figure the actual force (measured either directly from forces on individual surfaces or from the integrated stream thrust at vehicle exit plane e) is 15,045 N (see earlier sections). In comparison, the axial force on the vehicle as computed using the entropy analysis technique in Eq. (32) is then plotted vs selected size of the global stream tube (i.e., as shown on the axis, the ratio of the cross-sectional area of that stream tube, designated as A_{wake}, to the exit area associated with the vehicle exit plane, designated as $A_{\mathrm{veh-exit}}$). The computed force from Eq. (32), which is based purely on entropy considerations in the overall flowfield, exhibits rapid asymptotic approach to the (actual) force; thus, this plot verifies the theoretical analysis represented by Eq. (32). Similar results are also shown in Fig. 8 for the reversible axial force on the vehicle, that is, for the force experienced by the vehicle if all processes in the bounding stream tubes over and through the vehicle were reversible (from i to e). Here the reversible force as measured directly from forces or from the integrated stream thrust changes between e and i is 31,316 N (for the reversible flowfield). The entropy analysis as given by Eq. (32) (here the fluid dynamics associated with the reversible vehicle exit plane are used in the wake-mixing process) yields the entropy-generated overall vehicle force as shown, which again asymptotically approaches the value of 31,316 N for large cross-sectional sizes of the global stream tube. Also illustrated is the lost axial force (i.e., the difference between the reversible axial force and the actual axial force) as determined by Eq. (35). As is expected, the lost axial force as computed from the entropy method approaches the actual value of 16,271 N asymptotically at larger cross-sectional areas of the global stream tube. Asymptotic convergences at large area ratios for the wake entropy generation rates are shown in Fig. 9 for both actual and reversible vehicle flowfields. The irreversibility in the wake is generally much larger than the irreversibility associated with the vehicle; furthermore, the reversible vehicle itself has significant irreversibility in its wake as seen in Fig. 9.

VIII. Theoretical Framework for General Relationship of Vehicle Forces to Entropy Production Including Wake-Mixing Process

Preceding sections provided the relationship between overall axial vehicle force and entropy production across both vehicle and vehicle wake; this analysis was based on the assumption of thermodynamic heat interaction (Rayleigh heating process), constant specific heats, and no mass (fuel) addition. The expression for vehicle axial-force power that results is simple; vehicle axial-force power is seen to be a function of heat and work interactions between airframe and fluid and entropy production in the flow over (and through) the vehicle and in the wake-mixing process.

It is now necessary to similarly define the solution to the completely general problem, that is, to develop the analysis for the case of a vehicle with fuel mass addition to the fluid stream, fuel/air reaction (i.e., multiple species), non-constant specific heats throughout the flowfield, etc. The following analytical development for the relationship between overall vehicle axial force and entropy production in steady flow is, for ease of presentation, shown using quantities based on uniform flow (meaning uniform across a given cross section). Because this work ultimately involves the analysis of the flow across the global (vehicle + environment) stream tube (including the wake-mixing zone; see Fig. 7), this one-dimensionalized form is, in fact, directly applicable because the flow is indeed uniform at stream-tube inlet i and stream-tube exit w.

For the global stream tube as shown in Fig. 7, the magnitude of the overall axial force on the vehicle is properly defined as

$$F_{x(\mathrm{veh})} = \dot{m}_e u_w - \dot{m}_i u_i + P_w A_w - P_i A_i \tag{47}$$

This is the actual net axial force experienced by all wetted surfaces of the vehicle, which is embedded within the stream tube, external and internal (including fuel/propellant systems) (positive in the negative x direction). For the global stream tube chosen, in which all side boundaries extend well into the far field, $A_w = A_e = A_i$.

The vehicle has some fuel or propellant mass flow rate into the fluid from its structure, designated here as $\dot{m}_f$. Continuity across the global stream tube dictates that

$$\dot{m}_w = \dot{m}_e = \dot{m}_a + \dot{m}_f \tag{48}$$

Momentum considerations result in the following expression for the change in kinetic energy across the stream tube:

$$\dot{m}_w\left(u_w^2/2\right) - \dot{m}_i\left(u_i^2/2\right) = \dot{m}_w\left[(u_w - u_i)^2/2\right] - \dot{m}_f\left(u_i^2/2\right) - (P_w A_w - P_i A_i)u_i + F_{x(\mathrm{veh})}u_i \tag{49}$$

Energy considerations applied to the global stream tube result in the following:

$$\begin{aligned}\dot{m}_w\frac{u_w^2}{2} - \dot{m}_i\frac{u_i^2}{2} = {} & \dot{Q}_{\mathrm{flow-path}} + \dot{W}_{\mathrm{flow-path}} - \dot{m}_w\frac{\left(v_w^2 + w_w^2\right)}{2} \\ & + \dot{m}_f\frac{\boldsymbol{V}_{\mathrm{inj}}\cdot\boldsymbol{V}_{\mathrm{inj}}}{2} - \dot{m}_w\sum_{l=1}^{ncs}\alpha_{l,w}\left\{h_{0,l} + \int_{T_{\mathrm{ref}}}^{T_w} C_{p,l}\,\mathrm{d}T\right\} \\ & + \dot{m}_i\sum_{l=1}^{ncs}\alpha_{l,i}\left\{h_{0,l} + \int_{T_{\mathrm{ref}}}^{T_i} C_{p,l}\,\mathrm{d}T\right\} \\ & + \dot{m}_f\left\{h_{0(\mathrm{fuel})} + \int_{T_{\mathrm{ref}}}^{T_{\mathrm{inj}}} C_{p(\mathrm{fuel})}\,\mathrm{d}T\right\}\end{aligned} \tag{50}$$

Here $\dot{Q}_{\text{flow-path}}$ and $\dot{W}_{\text{flow-path}}$ are the heat and work rates (positive to the flow) supplied to the fluid from the structure of the vehicle. They are not, however, inclusive of any heat and/or work associated with the fuel system (which are implicitly present in the fuel total enthalpy rate). Here v_w and w_w represent the out-of-axial velocity components at the exit of the global stream tube, that is, they are the y and z velocity components orthogonal to the relative wind vector (along with the overall axial force is defined).

Finally, the second law considerations for the global stream tube results in the following expression:

$$\dot{m}_w \sum_{l=1}^{ncs} \alpha_{l,w} \left\{ s_{0,l} + \int_{T_{\text{ref}}}^{T_w} C_{p,l} \frac{\mathrm{d}T}{T} - r_l \int_{P_{\text{ref}}}^{P_w} \frac{\mathrm{d}P}{P} - r_l \ln \eta_{l,w} \right\}$$
$$- \dot{m}_i \sum_{l=1}^{ncs} \alpha_{l,i} \left\{ s_{0,l} + \int_{T_{\text{ref}}}^{T_i} C_{p,l} \frac{\mathrm{d}T}{T} - r_l \int_{P_{\text{ref}}}^{P_i} \frac{\mathrm{d}P}{P} - r_l \ln \eta_{l,i} \right\}$$
$$= \dot{S}_{\text{vehicle}} + \dot{S}_{\text{wake}} + \dot{S}_{\text{inj}} \tag{51}$$

and

$$\dot{S}_{\text{inj}} = \dot{m}_f \left\{ s_{0\text{fuel}} + \int_{T_{\text{ref}}}^{T_{\text{inj}}} C_{\text{pfuel}} \frac{\mathrm{d}T}{T} - r_{\text{fuel}} \int_{P_{\text{ref}}}^{P_{\text{inj}}} \frac{\mathrm{d}P}{P} \right\} \tag{52}$$

$\dot{S}_{\text{vehicle}}$ is the total entropy generation rate from freestream i to vehicle exit plane e and includes all entropy generated as a result of irreversibilities (but excluding entropy generated within the fuel system prior to injection, which is implicit in $\dot{S}_{\text{inj}}$, the entropy flow rate associated with the fuel injection) as well as entropy associated with heat transfer from the vehicle structure to the fluid. $\dot{S}_{\text{wake}}$ is the entropy generation caused by the wake-mixing process behind the vehicle.

Recall that the vehicle is embedded within the global stream tube, which itself must extend into the far field on the side boundaries in order to adequately encompass the wake-mixing process. In the proper limit of (very) large cross-sectional area defining this stream tube, then, the effects of the vehicle at the wake exit simply become a perturbation on the freestream conditions, that is, the following can be written:

$$u_w = u_i + \Delta u, \qquad T_w = T_i + \Delta T, \qquad P_w = P_i + \Delta P$$
$$\alpha_{lw} = \alpha_{li} + \Delta\alpha_{l,i}, \qquad \eta_{lw} = \eta_{li} + \Delta\eta_{l,i} \tag{53}$$

where Δu, ΔT, ΔP, $\Delta\alpha$, $\Delta\eta \to \varepsilon$ as $A_w \to \infty$.

Rigorously, the wake-mixing process should be computed assuming equilibrium chemical composition of the products at wake exit (although keep in mind that the composition will asymptotically approach that of freestream air as the stream tube is increased in size to encompass the wake mixing). However, because the temperature at the exit of the wake T_w is very close to the freestream temperature T_i, which is always very low from the standpoint of chemical reactions (i.e., 230 K at 30 km altitude for a typical application), the products can be assumed with confidence to be those associated with complete reaction at wake exit; hence, equilibrium analysis need not be done. This approximation greatly eases the complexity of the wake-mixing process for the general case; examples done following this section demonstrate the validity of the complete reaction approximation for wake exit.

These relationships and equations can now be analytically combined within the given constraints (perturbations from freestream) to yield an expression for the vehicle axial force power, that is, $F_{x(\text{veh})} \cdot u_i$, given here:

$$F_{x(\text{veh})} \cdot u_i = \dot{Q}_{\text{flow-path}} + \dot{W}_{\text{flow-path}} + \dot{m}_f \frac{u_i^2}{2} + \dot{m}_f \frac{\boldsymbol{V}_{\text{inj}} \cdot \boldsymbol{V}_{\text{inj}}}{2}$$
$$+ \dot{m}_f \left\{ h_{0(\text{fuel})} + \int_{T_{\text{ref}}}^{T_{\text{inj}}} C_{\text{p(fuel)}} \, \mathrm{d}T \right\}$$
$$- \dot{m}_w \sum_{l=1}^{ncs} \alpha_{l,w} [h_{l,i} - T_i s_l(T_i, P_i, \eta_{l,w})]$$
$$+ \dot{m}_i \sum_{l=1}^{ncs} \alpha_{l,i} [h_{l,i} - T_i s_l(T_i, P_i, \eta_{l,i})]$$
$$- T_i (\dot{S}_{\text{vehicle}} + \dot{S}_{\text{wake}} + \dot{S}_{\text{inj}}) \tag{54}$$

where

$$h_{l,i} = h_{0,l} + \int_{T_{\text{ref}}}^{T_i} C_{P,l} \, \mathrm{d}T$$

$$s_l(T_i, P_i, \eta_l) = s_{0,l} + \int_{T_{\text{ref}}}^{T_i} C_{P,l} \frac{\mathrm{d}T}{T} - r_l \ln \frac{P_i}{P_{\text{ref}}} - r_l \ln(\eta_l)$$

This expression becomes exact in the limit of large A_w. For a thermally balanced vehicle with no (net) work interactions and a fuel temperature in tank of T_{tank}, Eq. (54) results in the following:

$$F_{x(\text{veh})} \cdot u_i = \dot{Q}_{\text{flow-path}} + \dot{m}_f \left\{ \frac{u_i^2}{2} + h_{0(\text{fuel})} + \int_{T_{\text{ref}}}^{T_{\text{tank}}} C_{\text{p(fuel)}} \, \mathrm{d}T \right\}$$
$$- \dot{m}_w \sum_{l=1}^{ncs} \alpha_{l,w} [h_{l,i} - T_i s_l(T_i, P_i, \eta_{l,w})]$$
$$+ \dot{m}_i \sum_{l=1}^{ncs} \alpha_{l,i} [h_{l,i} - T_i s_l(T_i, P_i, \eta_{l,i})]$$
$$- T_i (\dot{S}_{\text{vehicle}} + \dot{S}_{\text{wake}} + \dot{S}_{\text{inj}}) \tag{55}$$

Furthermore, for H_{fuel} being the heating value of the fuel, one can show the following from Eq. (55):

$$F_{x(\text{veh})} \cdot u_i = \dot{Q}_{\text{flow-path}} + \dot{m}_f \left(u_i^2 / 2 + H_{\text{fuel}} \right)$$
$$- T_i \left[\dot{S}_{\text{irr(vehicle)}} + \dot{S}_{\text{wake}} \right] \tag{56}$$

IX. Conclusions

A methodology that allows the calculation and analysis of the impact of irreversibility on high-speed aerospace vehicle forces (both axial and out of axial) is presented. This methodology is based on a basic control-volume/stream-tube-based approach to the problem and is rooted in the development of the understanding of the fundamental drivers on forces experienced by a vehicle in fluid flow. Using this theory as a basis, the capability for a complete vehicle-level auditing of forces is suggested and is then demonstrated in terms of irreversibility, energy interactions, and overall flow turning/area changes. This is integrated into a unified framework that provides the ability to exhaustively audit vehicle performance in terms of irreversibilities and energy interactions. A highly simplified high-speed vehicle example, which has both external and internal fluid dynamics is constructed. However, the methodology can be rigorously applied to any level of modeling.

The fundamental link between vehicle (overall axial) force and the irreversibilities and energy interactions occurring over (or through) the vehicle and the irreversibility occurring in the wake-mixing process is then explained and demonstrated analytically for constant specific heats and Rayleigh-type heat. Specifically, conventional exergy (or availability) is directly related to the vehicle axial-force development. This, however, mandates consideration and inclusion of the wake-mixing process in the analysis procedure. Vehicle axial-force development is related directly to the entropy production over and behind the vehicle, and optimization (under constraints) is seen to result in terms of maximum thrust-drag production at the condition of minimum overall entropy production. The methodology is verified utilizing the same simplified example defined for the performance-based audit; this example shows that the vehicle performance can be computed purely from entropy considerations. The analysis is then expanded for the completely general problem in which there is fuel mass addition, chemical reaction, variable specific heats, and multiple species, etc.

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